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Reinsurance as a Method of Managing the Risks of an Insurance Company's Bankruptcy

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ABSTRACT

Objective. To analyze strategies for reducing the risks of insurance company insolvency based on proportional and non-proportional reinsurance. The levels of such risks are assessed annually using probabilities that the insurance claims filed with the company exceed the amount of its accumulated insurance fund. **Methodology.** The study employs analytical modeling based on probabilistic assumptions, where insurance claims are treated as independent and identically distributed random variables. Mathematical expectation and variance are used to derive solvency conditions under proportional and non-proportional reinsurance schemes. **Results.** The practical feasibility of applying these types of reinsurance is discussed, and analytical equations are derived to describe the patterns of reducing the probability of insolvency for the primary insurance company (cedent) and increasing its insurance fund, depending on its chosen retention levels and the risk surcharges of both the cedent and the reinsurer. **Conclusion.** The considers single-criterion and dual-criterion optimization problems

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for these relationships, based on the minimization of the cedent's insolvency probability and/or the maximization of its insurance fund growth, subject to relevant constraints.
KEY WORDS: Insurance, Reinsurance, Insurance fund, Insolvency probability, Claims distribution.

Introduction

Insurance companies provide financial protection to organizations and individuals against losses that may arise due to various adverse events affecting their activities. However, these companies themselves are exposed to the risk of failing to meet their obligations. This risk, for each company over a given time interval (e.g., a year) ($t, t+1$) is characterized by the probability of technical insolvency, meaning the probability that the total amount of insurance claims filed during this period exceeds the accumulated insurance reserves of the company.

Such a situation does not necessarily imply that the company will be unable to continue operating in the insurance market, as it may replenish its financial resources by securing loans against future revenues or utilizing other financial mechanisms (Tikhomirov et al., 2023). However, the occurrence of such situations indicates financial instability, negatively affecting the company's competitiveness, which may ultimately lead to its actual insolvency (Mishina et al., 2019; Bressan, 2023).

Given this, every insurance company aims to reduce the risk of technical insolvency through more efficient business organization, particularly by utilizing reinsurance mechanisms (Mavrulova, 2021; Polyakova, Polyakov, 2021). Reinsurance enhances the financial stability of the primary insurance company (cedent) by transferring a portion of the claims it receives to another company (reinsurer) in exchange for a fee.
When structuring reinsurance, it is important to consider that while risk transfer reduces the probability of the cedent's technical insolvency, it also slows the growth of its insurance

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reserves, as part of the cedent's premium is paid to the reinsurer. Finding a balance in this trade-off involves determining "equilibrium" relationships between the amount of claims transferred to the reinsurer and the associated reinsurance premiums. The solution to this problem depends on the form of reinsurance.

So, the purpose of the article was to explore approaches to deriving such solutions for quota-share proportional and non-proportional excess-of-loss reinsurance.

Methodology

To simplify the calculations, we will assume that the cedent provides insurance services to N policyholders, each of whom is characterized by independent and identically distributed ($t, t+1$) insurance claims *x* with mathematical expectations $M[x]$ and variances σ_x^2. At the same time, the amount of the insurance premium π, paid by one policyholder to the cedent, is proportional to the mathematical expectation of payments:

$$\pi = (1+\gamma) \cdot M[x], \quad (1)$$

where γ – is the relative insurance surcharge.

In quota-share proportional reinsurance, the cedent retains part of the premium α (for own retention), $0 \le \alpha \le 1$, paying the reinsurer a premium with a risk surcharge γ_* (Dadkov, Turbina, 2007; Logvinova, 2010). At the same time, the amounts of payments to the policyholder are also distributed between the cedent and the reinsurer in proportion to α and $(1-\alpha)$. Usually, $\gamma_* \ge \gamma$, which ensures the attractiveness of reinsurance for the reinsurer. As a result, the

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amounts received in the funds of the cedent (π_c) and the reinsurer (π_r) in the interval ($t, t+1$) are determined by the following equations:

$$\pi_c = (1+\gamma) \cdot \alpha \cdot M[x] \cdot N,$$
$$\pi_r = (1+\gamma_*) \cdot (1-\alpha) \cdot M[x] \cdot N. \quad (2)$$

Considering the share of own retention α, the random amounts of payments by the cedent and the reinsurer in the interval ($t, t+1$) can be represented as follows:

$$V_c = \alpha \cdot V,$$
$$V_r = (1-\alpha) \cdot V \quad (3)$$

where V – is the total amount of payments to N policyholders in the interval ($t, t+1$), which is a random variable characterized by mathematical expectation $M[V] = V[x] \cdot N$ and variance $\sigma_V^2 = N \cdot \sigma_x^2$.

The change in the amount of the cedent's insurance fund in the interval ($t, t+1$) considering equations (1)-(3) will be determined by the following equation:

$$F^{t+1} = F^t + (1+\gamma) \cdot M[x] \cdot N - (1+\gamma_*) \cdot (1-\alpha) \cdot M[x] \cdot N - \alpha \cdot V, \quad (4)$$

where F^t – the amount of the cedent's insurance fund accumulated by time *t*.

Considering (4) the probability of the cedent's technical insolvency in the interval ($t, t+1$) is determined by the following condition (Edakov, 2001; Kalashnikov, Konstantidis, 1996):

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$$P(F^{t+1} < 0) = P(V > [(1+\gamma_*) \cdot M[x] \cdot N + \frac{F^t + (\gamma - \gamma_*) \cdot M[x] \cdot N}{\alpha}]) = \int_z^{\infty} f(V) dV, \quad (5)$$

where z – denotes the value contained in the right-hand side of inequality (5); $f(V)$ – probability density function of the total insurance payments.

Equations (4) and (5) indicate that the probability of the cedent's technical insolvency with a change in own retention α from 1 (no reinsurance) to 0 (full reinsurance) in the interval ($t, t+1$) decreases from value $P(V > [(1+\gamma) \cdot M[x] \cdot N + F^t])$ to zero. However, at the same time, the expected increase in its insurance funds due to the reduction in insurance premiums also decreases by the amount:

$$\Delta^1 F^t = (\gamma - (1-\alpha) \cdot \gamma_*) \cdot M[x] \cdot N \quad (6)$$

For $\alpha = 1$ (no reinsurance) and for $\alpha = 0$ (full reinsurance), the values of this indicator are respectively:

$$\Delta^1 F^t (\alpha = 1) = \gamma \cdot M[x] \cdot N,$$
$$\Delta^1 F^t (\alpha = 0) = (\gamma - \gamma_*) \cdot M[x] \cdot N. \quad (7)$$

Note that the value of the second indicator in (7) at $\gamma < \gamma_*$ becomes negative, i.e. the amount of the cedent's insurance fund in the case of full reinsurance decreases even in the absence of payments for insurance events.

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In non-proportional reinsurance based on excess of loss, payments to policyholders between the cedent and the reinsurer are distributed as follows. The cedent, upon receiving a claim, pays an amount not exceeding the amount of its own retention u. If the payments exceed this amount, the excess is covered by the reinsurer. Such a distribution of payments for an insured event is reflected in the following equation (Burkov et al., 2001; Dadkov, Turbina, 2007; Sichka, 2003):

$$V_c = min(x, y),$$
$$V_r = max(x - y, 0) \quad (8)$$

where V_c and V_r – the amounts of payments by the cedent and the reinsurer for one insured event, respectively.

In such a situation, the unconditional average amounts of payments by the cedent to the policyholder are determined as their mathematical expectations over the interval ($0, y$), and the reinsurer's – over the interval (y, ∞). These indicators, given a known probability density distribution of payments, can be estimated based on the following:

$$M[V_c] = \int_0^y xf(x)dx + \int_y^\infty f(x)dx,$$
$$M[V_r] = \left[\frac{\int_y^\infty xf(x)dx}{\int_y^\infty f(x)dx} - y\right] \cdot \int_y^\infty f(x)dx = \int_y^\infty xf(x)dx - y\int_y^\infty f(x)dx \quad (9)$$

where $f(x)$ – the probability density function of payments for one policyholder's claim.
The second expression in formula (9) represents the mathematical expectation of the difference between the average amount of payments in the interval (y, ∞) and the limit of own retention *y*. Obviously, the sum of the mathematical expectations of payments by the cedent and the

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reinsurer for one insured event equals the average amount of payments to the policyholder $M[V] = M[V_c] + M[V_r]$.

The total amount of payments by the cedent in the interval ($t, t+1$) is determined as the sum of claims from policyholders not exceeding the limit of own retention *y*:

$$S_c^t = \sum_{i=1}^{N} min(x_i, y) \tag{10}$$

where x_i – the amount of the claim from the i-th reinsurer. This indicator is a random variable.

Considering the proportionality condition of the premiums received by the cedent and the reinsurer, to the mathematical expectations of their independent payments, the amount of the cedent's insurance fund now $t+1$ is determined by the following equation:

$$F^{t+1} = F^t + (1+\gamma) \cdot M[V] \cdot N - (1+\gamma_*) \cdot M[V_r] \cdot N - S_c^t \tag{11}$$

Considering that $M[V] = M[V_c] + M[V_r]$ (11) could be presented in following form:

$$F^{t+1} = F^t + (\gamma - \gamma_*) \cdot M[V_r] \cdot N + (1+\gamma) \cdot M[V_c] \cdot N - S_c^t \tag{12}$$

Based on (12) the probability of the cedent's technical insolvency can be represented as the probability of the following event:

$$P(F^{t+1} < 0) = P(S_c^t > F^t + (\gamma - \gamma_*) \cdot M[V_r] \cdot N + (1+\gamma) \cdot M[V_c] \cdot N) \tag{13}$$

where the mathematical expectations of payments by the cedent and the reinsurer for one insured event ($M[V_c]$ and $M[V_r]$ and respectively) are defined by (9).

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With a large number of policyholders and the same distribution of payments under their contracts, it can be expected that the random variable $S_{ц}^{t}$ is normally distributed with a mathematical expectation $N \cdot M[V_{ц}]$ and variance estimated according to the following:

$$\sigma_S^2 = N \cdot \sigma_{V_c}^2 \quad (14)$$

where $\sigma_{V_c}^2 = \sum_{i=1}^{N} \frac{(min(x_i, y) - M[V_c])^2}{N}$ (15)

In such a situation, the probability of the cedent's technical insolvency can be estimated as the probability of exceeding the amount of its payments S_c^t, value z, $z = F^t + (\gamma - \gamma_*) \cdot M[V_r] \cdot N + (1 + \gamma) \cdot M[V_c] \cdot N)$:

$$P(S_c^t > z) = \int_z^{\infty} f(V_c) dV_c \quad (16)$$

where $f(V_c)$ – probability density function of the normal distribution of the cedent's payments with parameters and $M[V_c] \cdot N$ and σ_S^2.

In practice, for an arbitrary law of distribution of the cedent's payments, the probability $P(S_c^t > z)$ can be estimated using simulation modeling methods (Tikhomirov et al., 2016). According to these methods, in the interval ($t, t+1$) *K experiments* are conducted, $j = \overline{1, K}$, in each of which, based on historical data on the levels of payments to policyholders x_i^t, $i = \overline{1, N}$, $t = 1, 2, ..., T$, their sets x_{ij}^t are randomly formed. Based on them, for each experiment, the total amount is

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determined $S_{uj}^{t}=\sum_{i=1}^{N}x_{ij}^{t}$. Then, the probability of the cedent's technical insolvency in the interval ($t,t+1$) is determined for given levels of own retention *y*, risk surcharges γ and γ_{*}, the amount of the cedent's F^{t} as the ratio of the number of experiments in which the condition $S_{uj}^{t}>F^{t+1}$, was met to the total number of experiments *K*.

In the absence of reinsurance, the increase in the cedent's insurance fund in the interval ($t,t+1$) considering the conditions of $M[V_{n}]=0$ is determined based on equation (12) as:

$$F^{t+1}-F^{t}=\Delta F^{t}=(1+\gamma)\cdot M[x]\cdot N-S^{t} \tag{17}$$

where $M[x]$ – is the average claim size per policyholder; S^{t} – is the total amount of payments for policyholder claims (a random variable) in the interval ($t,t+1$).

"With full reinsurance, the increase in the cedent's insurance fund in the interval $(t,t+1)$ is determined using equation (12), taking into account the conditions $M[V_c]=0$, $S_c^t=0$:

$$\Delta F^{t}=(\gamma-\gamma_{*})\cdot M[x]\cdot N \tag{18}$$

For $\gamma<\gamma_{*}$ his indicator takes a negative value. Accordingly, in the absence of reinsurance, the probability of the cedent's technical insolvency can be estimated using the following:

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$$P(S^t > F^{t+1}) = P(S^t > (F^t + (1+\gamma) \cdot M[x])), \quad (19)$$

Whereas with full reinsurance ($y=0$, $S_c^t = 0$) this indicator equals zero.

Thus, under non-proportional reinsurance, the same dynamic effects on the probability of insolvency and the growth of the cedent's insurance fund are observed as in proportional reinsurance: as own retention decreases, the probability of the cedent's technical insolvency also decreases (a positive effect), but the growth of its insurance fund is reduced (a negative effect).

Results and discussion

Let us consider the conditions that determine, first, the feasibility of using reinsurance strategies to enhance the financial stability of insurance companies and, second, their effectiveness.

From equations (5) and (13), it directly follows that reinsurance is advisable when the company's accumulated insurance fund F^t is relatively small compared to the potential claim amounts within the interval ($t, t+1$). This situation may arise due to the company's limited initial capital or significant payments to claimants in previous periods, which prevented a substantial increase in its financial reserves. At the same time, the company may find it unacceptable to reduce the number of policyholders to lower claim payments, as such a step could lead to a decline in its competitiveness in the insurance market (Banin, 2020; Bressan, 2023).

An increased probability of technical insolvency may also occur when the number of policyholders is small, and their claims exhibit high volatility.

These conditions follow directly from the expression that defines the probability of an insurance company's technical insolvency (without reinsurance) under the assumption of a normal distribution of claim payments:

$$P(F^{t+1} < 0) = P(S^t > (F^t + (1+\gamma) \cdot M[x] \cdot N) - M[x] \cdot N) = 1 - \Phi(z), \tag{20}$$

where

$$z = \frac{F^t + (1+\gamma) \cdot M[x] \cdot N - M[x] \cdot N}{\sigma_{S^t}} = \frac{F^t + \gamma \cdot M[x] \cdot N}{\sqrt{N} \cdot \sigma_x};$$
$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{u^2/2} du;$$
$$\sigma_{S^t} = \sqrt{N} \cdot \sigma_x; \tag{21}$$

N – is the number of policyholders, γ – is the risk surcharge on the insurance premium, $M[x]$, σ_x^2 – are the mathematical expectation and variance of a single insurance claim, respectively.

Equation (20) indicates that the probability of an insurance company's technical insolvency decreases as the numerator decreases and the denominator of expression (21) increases, which confirms the conclusions stated above.

Let us illustrate these results with a hypothetical example that considers the dependence of the probability of technical insolvency on the number of policyholders. Suppose that policyholder claims against the insurance company in the interval $(t, t+1)$ follow a normal distribution with a mean claim amount of $M[x] = 1000$ monetary units and a standard deviation of $\sigma_x = 7100$,

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$\sigma_S^2 = \sigma^2(Nx) = 50 \cdot 10^6$. The number of policyholders is $N = 10$, and the risk surcharge is $\gamma = 0,4$. For simplicity, let us assume that $F^t = 0$. Substituting these initial data into (20), we obtain:

$$P(F^{t+1} < 0) = 1 - \Phi\left(\frac{8 \cdot 10^4}{\sqrt{10 \cdot 7100}}\right) \approx 0,039$$

When the number of policyholders doubles ($N = 20$) this probability decreases to approximately 0,005.

$$P(F^{t+1} < 0) = 1 - \Phi\left(\frac{8 \cdot 10^4}{\sqrt{20 \cdot 7100}}\right) \approx 0,005$$

Similar examples can easily be provided regarding the dependence of the probability of technical insolvency on the accumulated capital F^t and the standard deviation of claim payments σ_{S^t} .

In reinsurance, a key task is the coordination of insurance operation parameters, including the risk surcharges on the insurance premiums of both the cedent and the reinsurer γ and γ_*, respectively, as well as the cedent's retention limits *α* and *y*, ensuring both a reduction in the probability of the cedent's technical insolvency and a positive growth of its insurance fund through insurance premiums (Banin, 2020).

The condition for the growth of the cedent's insurance fund through insurance premiums under proportional reinsurance follows from the following relationship, derived from the right-hand side of equation (4):

$$(1+\gamma)\cdot M[x]\cdot N-(1+\gamma_*)\cdot(1-\alpha)\cdot M[x]\cdot N=[\gamma-(1-\alpha)\gamma_*]\cdot M[x]\cdot N>0. \quad (22)$$

From inequality (22) it follows that a growth of the cedent's insurance fund through insurance premiums ($\Delta^1 F^t > 0$) occurs when the following condition is met:

$$\alpha > (\gamma_* - \gamma)/\gamma_* . \quad (23)$$

The probability of the cedent's technical insolvency, in this case, decreases provided that the right-hand side of the inequality in equation (5) increases. Considering the relationship between the surcharges – $(\gamma_* > \gamma)$ this increase is achieved when the following condition holds:

$$F^t + (\gamma - \gamma_*)\cdot M[x]\cdot N > 0 , \quad (24)$$

or $$\gamma_* < \gamma + F^t / M[x]\cdot N \quad (25)$$

Expression (25) can be interpreted as a constraint on the size of the insurance surcharge for the reinsurer.

The growth of the cedent's insurance fund through insurance premiums under non-proportional reinsurance is determined by the following condition (see equation 12):

$$\Delta^1 F^t = (\gamma - \gamma_*)M[V_r]\cdot N + (1+\gamma)M[V_c]\cdot N > 0, \quad (26)$$

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where the mathematical expectations of insurance payments by the cedent $(M[V_n]$ and $M[V_ц]$) for a single insured event are defined by equation (9).

Given the obvious equality$M[V_r] = M[V] - M[V_c]$, where $M[V]$ – is the expected payout for a single insured event, equation (26) can be indicated as:

$$\Delta^1 F^t > 0 \Rightarrow ((1+\gamma_*)M[V_r] + (\gamma - \gamma_*)M[V]) > 0. \quad (27)$$

From inequality (27), it follows that the relationship between the surcharges γ and γ_*, ensuring a positive growth of the cedent's insurance fund through insurance premiums, takes the following form:

$$\frac{M[V_c]+\gamma M[V]}{M[V_r]} > \gamma_*. \quad (28)$$

Obviously, inequality (28) simultaneously defines the condition for reducing the probability of the cedent's technical insolvency in the interval ($t, t+1$). However, this inequality does not explicitly include the cedent's own retention *y*, which makes it somewhat difficult to justify its rational (or optimal) value. This drawback can be partially addressed by expressing the indicator$M[V_c]$ as a function of *y*, using the first equation in expression (9).

Taking into account the obtained constraints on the cedent's retention level over the time interval ($t, t+1$) and the parameters of insurance $(M[x], \sigma_x^2, N, \gamma)$ and reinsurance γ_* either a single-criterion optimization problem can be formulated to determine the optimal retention level for proportional and non-proportional reinsurance (*α* and *y* respectively) with the objective of minimizing the probability of the cedent's technical insolvency while maintaining a lower bound on the growth of its insurance fund, or a dual-criterion optimization problem can be posed, aiming to simultaneously minimize insolvency probability and maximize fund growth.

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Conclusion

The material presented in this study generally demonstrates that reinsurance is an effective mechanism for reducing the risk of an insurance company's insolvency over a specific time interval, particularly when the company has a relatively small insurance fund and an insufficient number of policyholders. However, reinsurance also leads to a reduction in the cedent's fund inflows, which may contribute to the ongoing risk of insolvency in the future.

In this context, a key challenge in structuring reinsurance is to justify the appropriate levels of premiums for both the cedent and the reinsurer, as well as the cedent's retention amount, considering its available financial resources and the statistical patterns of policyholder claim payments.

Addressing this challenge requires careful consideration of the relationships established in this study between these parameters, which define the conditions for reducing the probability of the cedent's technical insolvency and ensuring a growth of its insurance fund. Additionally, the study provides recommendations for formulating the optimization problem of the cedent's retention level within the constraints defined by these relationships.

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