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The St. Petersburg Paradox and Its Implications for Economic Decision-Making

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ABSTRACT

Objective. To examine the St. Petersburg Paradox from an economic perspective. The purpose of the study was to explore various theoretical resolutions, including

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expected utility theory, risk aversion, and behavioral economics, and their applications in modern economic decision-making. **Methodology.** The research employs a systematic literature review and comparative analysis of different approaches to resolving the St. Petersburg Paradox. Key methodologies include an evaluation of mathematical models, historical economic theories, and behavioral insights. The study also considers simulations and empirical data on risk perception to understand how individuals and markets respond to uncertain outcomes. **Results.** The analysis reveals that the paradox highlights the limitations of traditional expected value calculations in economic decision-making. **Conclusion.** The St. Petersburg Paradox remains a crucial concept in economic theory, illustrating the need for more sophisticated decision-making models beyond expected value maximization. While expected utility theory provides a widely accepted resolution, modern economic research continues to refine risk-based models to account for human behavior and real-world financial constraints.

KEY WORDS: St. Petersburg Paradox, Expected Utility Theory, Decision-making under uncertainty, Behavioral economics.

Introduction

History of the Paradox

The concept of the St. Petersburg Paradox originated from studies on gambling and probability theory. It was first discussed by Nikolai Bernoulli in his correspondence with the French mathematician Pierre Rémond de Montmort, but it was Daniel Bernoulli who provided a more detailed analysis in his Commentaries of the St. Petersburg Academy. Some researchers argue that the paradox was formulated by Leonhard Euler, who lived and worked in St. Petersburg for a significant period (Sekey, 2019).

The paradox emerged from the analysis of a game in which a player bets on the outcome of a coin toss. If the coin lands on heads, the player wins 2^n rubles, where n represents the number of tosses required to get heads. If the coin lands on tails, the player wins nothing.

The paradox lies in the fact that the expected value of the game is infinite, despite the very low probability of winning. This occurs because the expected value is calculated as the sum of the products of all possible winnings and their respective probabilities. In this case, the probability of getting heads on any given toss is 50%, and the expected value of the game is:

$$E = 2^1 * 0.5 + 2^2 * 0.5 + 2^3 * 0.5 + ...$$

This sum is infinite because each subsequent coin toss doubles the potential winnings. However, despite the infinite expected value, the probability of winning remains very low.

Daniel Bernoulli proposed a solution to the paradox by considering not only the expected value but also the degree of risk. He introduced the concept of utility, which accounts for both expected value and risk perception. This helped explain why individuals might be unwilling to participate in a game with infinite expected value, despite its mathematical appeal.

Thus, the St. Petersburg Paradox became a significant milestone in the development of probability theory and economics, demonstrating that expected value and risk must be considered together for a comprehensive understanding of human behavior under uncertainty.

The mathematical expectation of winnings in this game is infinite, as the sum of winnings grows geometrically at each step, while the probability of getting heads remains constant. This means that, theoretically, a player could expect to win an extremely large amount if they were to play the game an infinite number of times. However, this does not imply that a player should be willing to pay any price to participate. In reality, limited financial resources and the high cost of participation make it highly unlikely that a player would win a significant sum. The idea of paying an infinitely large amount to enter such a game is understandably irrational, which is why the problem is called a "paradox."

Thus, the essence of the St. Petersburg Paradox is that players are only willing to pay a small amount to participate in a game where the mathematical expectation of winnings is infinitely large.

Since in the St. Petersburg Paradox, only low-probability events yield high rewards, leading to an infinite expected value, this paradox itself suggests its resolution (Fedorov, 2019; Clark, 2002).

Various scholars, including Jean le Rond d'Alembert and John Maynard Keynes, rejected expected value maximization as a valid method for decision-making, even questioning its practical utility in such cases. Keynes argued that the relative risk of an alternative event occurring could be high enough to justify rejecting all choices leading to that event, even if the expected value of a positive outcome was exceptionally large (Menger, 1967; Szabó-Szentgróti et al., 2021).

For this reason, the St. Petersburg Paradox is often used as an example of how mathematical models may fail to align with real-world decision-making.

Example of a Coin Toss

The St. Petersburg Paradox illustrates the discrepancy between a player's theoretically optimal behavior and their intuitive decision-making.

Let's consider the following scenario:

A player places an initial bet and flips a fair coin (50% probability for each outcome). The game continues until heads appears, at which point the player receives a payout based on specific conditions:

1. If heads appears on the first toss, the player wins 2^0 (so, it's 1 ruble).

2. If heads appears on the second toss, the player wins 2^1 (so, it's 2 rubles).

3. If heads appears on the n-th toss, the total winnings amount to $2^{(n-1)}$ rubles.

Thus, with each additional toss, the payout doubles, following a sequence of powers of two: 1, 2, 4, 8, 16, 32, and so on.

The probability that the number of coin tosses in a given game exceeds a certain value t is given by $\left(\frac{1}{2}\right)^t$. If a player participates in no more than m games, the probability that the number of tosses will exceed n is $1-(1-\frac{1}{2^t})^m$.

Methodology

This study employs a systematic and objective approach to analyzing the St. Petersburg Paradox, incorporating a comprehensive review of academic literature, comparative analysis of different theoretical perspectives, and synthesis of findings. The research is based on a critical examination of sources that explore the paradox's implications for decision-making, risk assessment, and economic modeling. To ensure relevance and credibility, the selection of literature was guided by specific keywords, including "St. Petersburg Paradox" "decision-making", "risk," and "risk assessment". The study prioritized peer-reviewed articles and fully accessible texts to allow for an in-depth evaluation of various arguments and interpretations.

A thorough examination of existing research enabled the identification of key theoretical frameworks and practical implications. Comparative analysis was used to assess different resolutions to the paradox, from expected utility theory to alternative models of risk perception and bounded rationality. To illustrate the findings, carefully selected case studies and research examples were incorporated, providing a well-rounded perspective on the paradox's significance in economics and probability theory.

This approach ensures that both mathematical reasoning and behavioral insights are considered in understanding the paradox and its broader applications.

Results

Key Concepts and Approaches to Resolving the St. Petersburg Paradox

Several approaches have been proposed to address the St. Petersburg Paradox, some of which seek to explain why its expected value is infinite, while others explore how the concept of expected value can be applied in real-world decision-making (Feller, 2019; Friedman, Savage, 2019; Nelyubina, 2015; Diev, 2014).

One of the most widely accepted explanations is expected utility theory, introduced by Daniel Bernoulli. This approach suggests that players should choose strategies based on their personal preferences and constraints rather than relying solely on expected value. Bernoulli argued that the value of money is subjective, meaning that people derive less utility from each additional unit of wealth. As a result, a small but highly probable win may be more valuable to a player than a large but rare reward.

Another perspective considers the economic theory behind the paradox. From this viewpoint, players make decisions based on market expectations and external conditions. In the case of the St. Petersburg Paradox, the willingness to participate in the game depends on the potential changes in the cost of entry, which may fluctuate due to supply and demand in financial markets.

A third approach applies statistical theory by simulating the game over many trials. This method evaluates which strategies lead to the most favorable outcomes on average. It suggests that rational players should base their decisions on the probabilities of winning and losing, optimizing their strategy accordingly.

Another interpretation comes from bounded rationality and decision-making under uncertainty. According to this view, a player may be willing to pay a high price for entry if they perceive the potential winnings as critical to their goals. For example, if a person has limited financial resources and is in a high-stakes situation, they might accept greater risks despite the paradoxical nature of the game.

Despite these various explanations, no single solution fully resolves the paradox, making it a continuing topic of interest in probability theory, economics, and game theory. One possible way to address the paradox is by modifying the game's rules – for instance, limiting the number of rounds a player can play or placing a cap on winnings. Such adjustments significantly reduce the expected value, making the game more realistic and attractive to players.

Another practical resolution considers the cost of participation. If players must pay an entry fee, their expected winnings must be high enough to justify the expense. If the cost of playing exceeds the expected payout, rational players will choose not to participate, reducing the paradox's theoretical impact (Kudryavtsev, 2013; Kraitchik, 1942; Bernoulli, 1993).

Thus, while the St. Petersburg Paradox may appear to be a simple gambling scenario, it has far-reaching implications in probability theory and economics. Addressing the paradox requires consideration of both mathematical principles and behavioral factors, including individual preferences, market dynamics, and the cost of participation.

Expected Value and Utility

The expected value of a game's winnings or losses can be calculated using the following formula:

$$E(x)=p_1x_1+p_2x_2+\ldots+p_nx_n,$$

where p_1, p_2,….. p_n – are the probabilities of each outcome,

x_1, x_2, …x_n – are their corresponding values.

In the case of the St. Petersburg game, the expected value of a player's winnings is infinite:

$$\Sigma = \frac{1}{2} * 1 + \frac{1}{4} * 2 \frac{1}{8} * 4 \frac{1}{16} * 8 + \cdots = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots = \infty$$

This infinite sum suggests that a rational player should be willing to pay any price to participate. However, most individuals are unwilling to do so, revealing a gap between mathematical expectation and actual decision-making behavior. Daniel Bernoulli proposed that people do not simply maximize potential monetary gains but instead seek to maximize their expected utility, which accounts for diminishing marginal returns of wealth. According to this idea, additional wealth brings less and less additional satisfaction, which explains why individuals do not treat all monetary gains equally.

For example, imagine a scenario where a friend owes you 100 rubles but instead offers to play a coin toss game: If you win, you get 200 rubles (earning an extra 100 rubles); If you lose, you receive nothing.

From a mathematical expectation perspective, the expected value of this bet is zero, making it neither favorable nor unfavorable. However, some individuals may choose to play, hoping for a higher reward, while others may refuse to take the risk and demand their original 100 rubles.

Similarly, consider a roulette bet: Suppose a player has 10,000 rubles and decides to bet 5,000 rubles on black. If they lose, they end up with 5,000 rubles, but if they win, their total increases to 15,000 rubles.

In this case, expected utility is calculated as:

$$U = \sum_{i-1}^{n} Pi * (Xi)$$

where: *pi* – represents the probability of each outcome,

xi – is the utility of that outcome.

If winning and losing are equally probable, the expected utility may actually be negative due to diminishing marginal utility, making the bet less attractive (Feller, 2019).

The utility function allows for an assessment of how important a particular gain is to an individual. For example, if a 100,000-ruble win increases the utility function of a poor person more significantly than that of a billionaire, then the expected utility of this gain will be higher for the poor person. A logarithmic utility function is often used to account for diminishing marginal utility. For instance, if a 2^{n-1} ruble win follows a logarithmic utility function, then the expected utility remains finite, despite the infinite mathematical expectation.

The expected utility theory was first published in the second edition of "Theory of Games and Economic Behavior" by John von Neumann and Oskar Morgenstern (1967) and was initially introduced as a supplement to game theory. In its original form, the authors extended game theory by incorporating the axioms of expected utility. Expected utility theory itself serves as a mathematical foundation for analyzing and predicting the behavior of rational agents under risk and uncertainty.

Although expected utility theory has been established for a long time, it is still actively used today in various fields of decision-making. In finance, for instance, investors apply the theory to evaluate portfolio selection, particularly when constructing portfolios that balance risk and return. In economic planning, the theory can be utilized by organizations or governments to assess various political and investment projects, considering standard planning risks.

However, expected utility theory has faced significant criticism and limitations, even in stable conditions. The theory assumes that a decision-maker possesses complete information about the probabilities and utilities of all possible outcomes – an assumption that rarely holds.

In conclusion, while expected utility theory remains fundamental in studying decision-making under risk, its "pure form" is often impractical for modern risk assessment due to the rapidly growing number of influencing factors. The theory lacks the flexibility to fully account for these factors, making it more suitable as a research framework rather than a definitive decision-making manual in today's world (Schumpeter, 2001; Cowen, High, 1988).

Cramer's Approach

Gerhard Cramer further developed the analysis of the St. Petersburg Paradox and proposed an alternative perspective on its resolution. He emphasized that the infinite expected value of the game does not necessarily reflect real-world expectations, as individuals tend to avoid risk and often perceive potential losses as more significant than potential gains.

Let's recall the example where, in a coin toss game, the payout is $2^{(n-1)}$ rubles, where n is the number of flips required to obtain heads for the first time.

Cramer argued that no rational person would be willing to pay a large sum to participate in this game. He proposed that, in reality, people evaluate money based on the utility they can derive from it rather than its absolute amount. If a financial gain exceeds a certain threshold, its marginal utility decreases, as there may be fewer valuable ways to spend the additional money.

Thus, the utility of money does not grow proportionally to monetary winnings but at a diminishing rate. Based on this observation, Cramer proposed a new utility function:

v(c) = c, where c – is the amount of money, a v(c) – represents the perceived utility of that amount for an individual.

In this case, the expected utility is calculated as::

$$0.5*1+0.25*2+0.125+\ldots.. = 1$$

From the perspective of utility, this situation appears differently than from a purely financial standpoint, leading to a reduction in risk appetite. This is precisely why we previously emphasized the importance of distinguishing between the expected monetary value of winnings and their expected utility.

Discussion

Resolving the Paradox: Limitations and Practical Considerations

The resolution of the St. Petersburg Paradox involves not only theoretical concepts but also practical constraints that must be considered.

For the game described in the paradox to take place, Player A would need to have unlimited financial resources. This is practically impossible, as even central banks can issue only a finite amount of money. If Player A had unlimited wealth, excessive payouts could lead to inflation, thereby reducing Player B's ability to receive large winnings.

Additionally, the coin toss process takes time. If the number of tosses is constrained by a time limit, the expected monetary value of the game becomes finite. Even if the flips were instantaneous, there remains a nonzero probability that the game could theoretically never end, making the infinite expectation purely theoretical rather than practical.

People also tend to misinterpret small probabilities, such as 0.01% vs. 0.0001%, which can lead them to overestimate the likelihood of extremely rare events. While individuals may be willing to take risks, there is often a threshold beyond which they refuse to participate (Nelyubina, 2015).

Expected utility theory further explains why individuals may reject games with infinite expected value. For instance, a $1,000 win may not be as significant to a wealthy person as a $100 win is to someone in poverty, meaning that the expected utility of the first win is lower. However, people don't always rationally assess the utility of potential winnings.

The Importance of the St. Petersburg Paradox in Modern Economic Theory

The St. Petersburg Paradox has significant implications for modern economic theory, particularly in game theory and decision-making. The paradox highlights that expected value can be an inadequate tool for describing risk and decision-making under uncertainty.

It also demonstrates how personal preferences and constraints influence players' choices, emphasizing the need to consider both mathematical and behavioral factors in decision-making. The paradox is particularly relevant in economic research related to valuing risky investments, such as stock markets and financial speculation. Understanding the relationship between expected winnings and risk perception can help investors make more informed financial decisions.

The paradox remains a subject of active research in economic theory and game theory, as its resolutions continue to have practical relevance in situations of uncertainty and risk (Vatnik, 2008; Weirich, 1984; Weber, 1998).

In recent years, the St. Petersburg Paradox has also been studied extensively in the context of gambling behavior. In his 2016 study, Decision Making and the St. Petersburg Paradox: Focusing on Heuristic Parameters, Considering the Non-Ergodic Context and Gambling Risks, Antonio Cappiello analyzed the irrationality of human decision-making and the misinterpretation of the paradox itself. Based on his observations, Cappiello provided practical insights into how players can avoid the trap of infinite expected value and consider the real-world limitations of gambling games with an infinite number of repetitions.

Modern Interpretations of the St. Petersburg Paradox and Perspectives from Different Scholars

Depending on their perspective on probability theory, different scholars interpret the St. Petersburg Paradox in different ways. Some view it as an example of misinterpreted mathematical expectations, demonstrating how they can be misleading. Others consider the paradox an interesting mathematical problem with no real-world implications.

There are also scholars who use the paradox as a foundation for discussions on how probabilities should be defined and how they can be applied in real-world decision-making. Some researchers propose alternative probability models, which they argue provide more accurate solutions.

Overall, while the St. Petersburg Paradox is widely recognized as an important topic in probability theory and mathematics, scholars differ in their views and approaches to resolving it (Carol, 1996).

Several modern interpretations of the paradox provide deeper insights into its role in economic theory:

1. The Paradox as an Example of the Asymmetry Between Expected and Real Value: This interpretation suggests that the paradox highlights the inadequacy of expected value as a decision-making tool under uncertainty. Instead, real-world decisions

should consider the true probability of each outcome and its subjective value to each player, which may vary based on personal preferences and constraints.

2. The Paradox as an Illustration of the Advantage of Players with Limited Resources: This perspective suggests that players with limited financial resources have a strategic advantage over wealthier players because they tend to adopt more conservative strategies that factor in risk and potential losses more carefully.

3. The Paradox as Evidence of the Limits of Economic Rationalism: According to this interpretation, the paradox demonstrates that economic rationalism has its limits in real-world decision-making, where people are influenced by emotions, biases, and non-economic factors. This underscores the importance of behavioral economics in studying financial decision-making.

4. The Paradox because of Imperfect Market Competition: Another interpretation attributes the paradox to market imperfections, where players do not always have equal access to opportunities and resources. This can lead to unequal distributions of winnings and losses, distortions in perceived value, and other negative consequences for economic welfare. This perspective highlights the importance of analyzing market structures and developing policies to reduce inequality and promote fair competition.

5. The Paradox as an Example of Risk and Loss Aversion: This interpretation suggests that the paradox can be used to explain phenomena such as loss aversion and risk perception. People are generally more inclined to take risks when they expect large rewards, but they may act more cautiously when there is a potential threat of losses. This can lead to distortions in how they evaluate probabilities and make decisions under uncertainty.

Conclusion

The St. Petersburg Paradox remains one of the most intriguing and controversial puzzles in the history of economics. It challenges traditional notions of rational behavior, demonstrating that individuals make decisions based not only on mathematical expectations but also on psychological and behavioral factors.

Throughout the analysis, key concepts and approaches proposed by Nikolai Bernoulli and Gerhard Cramer were examined. Bernoulli introduced expected utility theory, which accounts for not only the monetary expectation of winnings but also their perceived usefulness to the individual. Cramer, on the other hand, emphasized the importance of psychological and behavioral aspects in economic decision-making.

The application of utility theory helped explain why individuals may refuse to participate in games with infinitely high expected monetary values. The marginal utility of each additional unit of income decreases as overall wealth increases, making the expected utility of winnings finite, even though the expected monetary value remains infinite.

However, despite theoretical explanations, practical factors also play a crucial role. Financial limitations, time constraints, and psychological biases can significantly impact decision-making. People tend to misinterpret small probabilities and exhibit risk tolerance only up to a certain point.

In conclusion, the St. Petersburg Paradox serves as an important lesson for economists and researchers, highlighting the necessity of a comprehensive approach in analyzing economic decisions. It demonstrates how mathematical models can be used to analyze risk and decision-making across various contexts, while also emphasizing the value of critical thinking and analytical reasoning in identifying logical inconsistencies and contradictions within such models.

References

Bernoulli, D. (1993). *An attempt at a new theory of measuring risk*. In Theory of consumer demand (pp. 11–27). Moscow: Economic School.

Carol, A. (1996). *Financial Risk Management and Analysis*. New Jersey: John Wiley & Sons.

Clark, M. (2002). *The St. Petersburg paradox*. In Paradoxes from A to Z. London: Routledge.

Cowen, T., & High, J. (1988). Time, bounded utility, and the St. Petersburg paradox. *Theory and Decision*, 25(3), 219–223. https://doi.org/10.1007/BF00133163

Diev, V. S. (2014). The problem of choice and decision-making in an interdisciplinary context. *Bulletin of Tomsk State University. Philosophy. Sociology. Political Science*, 2(22).

Fedorov, D. A. (2019). *One of the ways to resolve the St. Petersburg paradox*. In A. A. Ovodenko (Ed.), Current problems of the economy of modern Russia: Collection of scientific papers (Vol. 5, pp. 368–370). GUAP.

Feller, W. (2019). *An introduction to probability theory and its applications*. Moscow: Mir.

Friedman, M., & Savage, L. J. (2019). *Utility analysis in choice among risky alternatives*. In V. M. Galperin (Ed.), Theory of consumer behavior and demand (Vol. 1, pp. 208–249). Saint Petersburg: Economic School.

Kraitchik, M. (1942). *The Saint Petersburg paradox. In Mathematical recreations*. New York: W.W. Norton.

Kudryavtsev, A. A. (2013). The St. Petersburg paradox and its significance for economic theory. *Bulletin of St. Petersburg State University*, 5(3).

Menger, K. (1967). *The role of uncertainty in economics*, in: Essays in selection: efficient diversification of investment. New York: John Wiley & Sons.

Nelyubina, M. S. (2015). Non-standard problems of probability theory. *Scientific and Methodological Electronic Journal "Concept"*, 25.

Neumann, J. von, & Morgenstern, O. (1970). *Theory of games and economic behavior*. Moscow: Nauka.

Schumpeter, J. (2001). *History of economic analysis*. Saint Petersburg: Economic School.

Sekey, G. (2019). *Paradoxes in probability theory and mathematical statistics*. Moscow: Mir.

Szabó-Szentgróti, G., Végvári, B., & Varga, J. (2021). Impact of industry 4.0 and digitization on labor market for 2030-Verification of Keynes' Prediction. *Sustainability*, 13(14). https://doi.org/10.3390/su13147703

Vatnik, P. A. (2008). Daniel Bernoulli as an economist. *Finance and Business*, (2), 188–194.

Weber, C. E. (1998). The St. Petersburg paradox: A resolution for impatient risk seekers. *International Advances in Economic Research*, 4, 367–373.

Weirich, P. (1984). The St. Petersburg gamble and risk. *Theory and Decision*, 17, 193–202.